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Hannay's angle and Berry's phase for $SU(1,1)$ systems

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Abstract. The classical and quantum systems related to the group $SU(1,1)$ are considered. The Hannay angle is calculated and its relation to Berry's phase established.

1. Introduction

It is now well known [1] that Berry's phase possesses, for integrable systems, a classical counterpart called Hannay's angle. This consists of an extra shift picked up by the angle variables of the system as the parameters undergo a closed adiabatic excursion. Berry [2] established the precise relationship between Berry's phase in the semiclassical limit and Hannay's angles (for mathematically rigorous treatment see [3]). His proof applies to the quantum systems obtained by quantizing the classical systems with the standard phase space (q,p). The quantum systems with a finite number of levels ('spin' systems) were considered by Gozzi and Thacker [4] and Giavarini *et al* [5]. The strategy was to show that a quantum system with a finite number of levels has a straightforward classical counterpart in terms of Grassmann variables. The Hannay angles for these classical Grassmannian systems were then calculated and compared with the corresponding Berry's phase. The result agreed with Berry's semiclassical formula. Such an analysis, although very elegant, has a slightly formal character because the classical Grassmannian system can hardly be viewed as a classical limit of the corresponding quantum one. Maamache *et al* [6] presented a very nice analysis of Berry's relation, establishing a direct link between the quantum and classical transports. The main observation was that the proper tool for describing the semiclassical states is provided by the coherent states. By exploiting this idea the authors of [6] were able to give a straightforward derivation of the relation between the classical and quantum aholonomies. The generality and elegance of this approach allows one to treat the systems with a finite number of levels, which have no standard classical counterpart. Inspired by this paper, we considered [7] the simplest system with a finite number of levels, the quantum spin in an external magnetic field. In order to analyse the link between Berry's phase and Hannay's angle we have used the general approach to the problem of a classical limit for quantum systems, developed in a nice paper by Yaffe [8] and based on some earlier ideas due to Berezin [9]. The main ingredients of this approach are the coherent states and the concept of quantization on coadjoint orbits.

In the present paper we consider the simplest non-compact Lie group—$SU(1,1)$. In section 2 we describe very briefly the standard symplectic structure on the coset space

$SU(1,1)/U(1)$. Given this structure and an arbitrary classical Hamiltonian the relevant action-angle variables are introduced and the formula for Hannay angle (equation (24)) is derived for the family of Hamiltonians obtained by the action of $SU(1,1)$ on some initial Hamiltonian. It takes an especially simple form if the initial Hamiltonian is invariant under the $U(1)$ subgroup. In section 3 the quantum dynamics based on unitary representations of $SU(1,1)$ (discrete series) is considered and, following Yaffe, the link with classical dynamics is established. The semiclassical relation between Berry's phase and Hannay's angle is shown to be here an exact one. This section is concluded with some remarks concerning the $SU(1,1)$ approach to the harmonic oscillator.

Let us conclude the introduction with a few remarks. We will be dealing with a generalized phase space and it is advantegous to write all formulae in such a way that all canonical variables are considered on equal footing. This can be achieved in any particular case. For example, consider the formula for the function generating the canonical transformation to the action-angle variables [10]

$$S(q,I)=\int_{q_0}^{q} p\,\mathrm{d}q \qquad p=\frac{\partial S}{\partial q} \qquad \varphi=\frac{\partial S}{\partial I}$$

here integration is along a path lying on invariant torus. This formula can be put in a form more symmetric with respect to q and p. Namely, if we define

$$\tilde{S}=S-\tfrac{1}{2}pq+\text{constant}$$

we can write

$$\tilde{S}(q,p,I)=\frac{1}{2}\int_{q_0}^{q}(p\,\mathrm{d}q-q\,\mathrm{d}p)$$

and

$$\frac{\partial\tilde{S}}{\partial q}=\tfrac{1}{2}p \qquad \frac{\partial\tilde{S}}{\partial p}=-\tfrac{1}{2}q \qquad \frac{\partial\tilde{S}}{\partial I}=\varphi\,.$$

Here the second equation is an identity which reflects the fact that q and p are constrained to lie on a torus. This is generalized as follows: we can view the generating function as depending on all initial canonical variables, keeping in mind that not all equations defining canonical transformation are then independent.

In what follows we consider the 'canonical' transformations which change the Poisson bracket. Although it is easy to modify accordingly Berry's derivation of Hannay's angles [2] we present here a slightly more straightforward derivation which has an obvious generalization in the 'non-standard' case. Let

$$\varphi=\varphi(q,p,X)$$

be the angle variable. An additional angular velocity related to the time dependence of X is equal to

$$\frac{\partial\varphi}{\partial X}\dot{X}\,.$$

But

$$\varphi=\frac{\partial S(q,I,X)}{\partial I}$$

and

$$\frac{\partial\varphi(q,p,X)}{\partial X}=\frac{\partial^2 S}{\partial I\,\partial X}+\frac{\partial^2 S}{\partial I^2}\frac{\partial I(q,p,X)}{\partial X}.$$

On the other hand, taking a derivative with respect to I, we have

$$0=\frac{\partial^2 S}{\partial^2 I}+\frac{\partial^2 S}{\partial I\,\partial q}\frac{\partial q(\varphi,I,X)}{\partial I}.$$

Taking the derivative of $p=\partial S(q,I,X)/\partial q$ with respect to X gives

$$0=\frac{\partial^2 S}{\partial q\,\partial I}\frac{\partial I(q,p,X)}{\partial X}+\frac{\partial^2 S}{\partial q\,\partial X}.$$

By combining the three last equations we arrive at the Berry formula, namely

$$\frac{\partial\varphi(q,p,X)}{\partial X}=\frac{\partial^2 S}{\partial I\,\partial X}+\frac{\partial^2 S}{\partial q\,\partial X}\frac{\partial q}{\partial I}=\frac{\partial}{\partial I}\left\{\frac{\partial S}{\partial X}(q(\varphi,I,X),I,X)\right\}.$$

2. Classical dynamics on $SU(1,1)/U(1)$†

In this section we consider classical dynamics on coset space $SU(1,1)/U(1)$.

The structure of the coset space $SU(1,1)/U(1)$ can be revealed by noting that the general element U can be written in the form

$$u=\frac{1}{\sqrt{1-|\zeta|^2}}\begin{pmatrix}1 & -\zeta\\ -\bar\zeta & 1\end{pmatrix}\begin{pmatrix}e^{i\omega} & 0\\ 0 & e^{-i\omega}\end{pmatrix}\qquad |\zeta|<1. \tag{1}$$

Therefore, the coset space $SU(1,1)/U(1)$ can be viewed as an open unit disc in the complex plane (Poincaré disc).

In order to study the geometry of $SU(1,1)/U(1)$ we need the Cartan forms. They read

$$\begin{aligned}\lambda&=i\frac{(\bar\zeta\,d\zeta-\zeta\,d\bar\zeta)}{1-|\zeta|^2}\\ \omega_+&=\frac{i\,d\zeta}{1-|\zeta|^2}\\ \omega_-&=-\frac{i\,d\zeta}{1-|\zeta|^2}.\end{aligned}\tag{2}$$

It is straighforward to check the validity of Cartan–Mauer equations

$$d\lambda=2i\omega_-\wedge\omega_+\qquad d\omega_\pm=\pm i\lambda\wedge\omega_\pm. \tag{3}$$

Next, the invariant 2-form Ω can be introduced

$$\Omega\equiv\omega_+\wedge\omega_-=\frac{1}{(1-|\zeta|^2)^2}d\zeta\wedge d\bar\zeta \tag{4}$$

which is closed, $d\Omega=0$, and non-degenerate. The symplectic structure introduced above allows us to define the Poisson bracket. Given any two functions $f=f(\zeta,\bar\zeta)$, $g=g(\zeta,\bar\zeta)$ we put

$$\{f,g\}\equiv\frac{(1-|\zeta|^2)^2}{ia}\left(\frac{\partial f}{\partial\zeta}\frac{\partial g}{\partial\bar\zeta}-\frac{\partial f}{\partial\bar\zeta}\frac{\partial g}{\partial\zeta}\right) \tag{5}$$

† All relevant concepts and results concerning $SU(1,1)$ are taken from the book by Perelomov [11].

where a is a real constant to be specifed later. For any real Hamiltonian $H = H(\zeta, \bar{\zeta})$ we can write the Hamiltonian equations of motion

$$\begin{aligned} \dot{\zeta} &= \{\zeta, H\} = \frac{(1-|\zeta|^2)^2}{\mathrm{i}a}\frac{\partial H}{\partial \bar{\zeta}} \\ \dot{\bar{\zeta}} &= \{\bar{\zeta}, H\} = -\frac{(1-|\zeta|^2)^2}{\mathrm{i}a}\frac{\partial H}{\partial \zeta}. \end{aligned} \tag{6}$$

These equations follow from the variational principle

$$\int\left(\frac{\mathrm{i}a}{2}\frac{\bar{\zeta}\,\mathrm{d}\zeta - \zeta\,\mathrm{d}\bar{\zeta}}{1-|\zeta|^2} - H(\zeta, \bar{\zeta})\,\mathrm{d}t\right) = 0. \tag{7}$$

Let us note that the first term in the integrand is basically the Cartan form λ; it plays the same role as $\frac{1}{2}(p\,\mathrm{d}q - q\,\mathrm{d}p)$ in standard dynamics: its exterior derivative gives the symplectic form.

The phase space of our system is two-dimensional. Therefore, for any conservative Hamiltonian $H(\zeta, \bar{\zeta})$ we obtain an integrable dynamics. Let us assume that the equation

$$H(\zeta, \bar{\zeta}) = E \tag{8}$$

defines a closed curve C in the phase space. The action I is defined by the equation

$$I = \frac{1}{2\pi}\oint_C \frac{\mathrm{i}a}{2}\frac{\bar{\zeta}\,\mathrm{d}\zeta - \zeta\,\mathrm{d}\bar{\zeta}}{1-|\zeta|^2} \tag{9}$$

which gives I as a function of E, $I = I(E)$. Also, if ζ_0 is a fixed point on C and $\zeta \in C$, we define the angle variable

$$\varphi \equiv \frac{\partial}{\partial I}\left\{\frac{\mathrm{i}a}{2}\int_{\zeta_0}^{\zeta}\frac{\bar{\zeta}\,\mathrm{d}\zeta - \zeta\,\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}\right\} \equiv \frac{\partial S(\zeta, \bar{\zeta}, I)}{\partial I} \tag{10}$$

where the integration is performed along C. It is easy to check that

$$\{\varphi, I\} = 1. \tag{11}$$

The (I, φ)-variables are the action-angle variables: I is a constant of motion while φ is a cyclic variable depending linearly on time.

The action of $SU(1,1)$ on $SU(1,1)/U(1)$ is given by

$$\zeta \to \zeta(\zeta, \eta, \omega) = \frac{\zeta \mathrm{e}^{2\mathrm{i}\omega} + \eta}{1 + \bar{\eta}\zeta\mathrm{e}^{2\mathrm{i}\omega}}. \tag{12}$$

It is easy to check explicitly that the Poisson bracket is invariant under this transformation.

Let us now consider the family of Hamiltonians given by the formula

$$H\left(\zeta, \bar{\zeta}; \eta, \bar{\eta}, \omega\right) = H\left(\zeta(\zeta; -\eta; -\omega), \bar{\zeta}(\zeta; -\eta; -\omega)\right). \tag{13}$$

Due to the invariance of symplectic structure we can immediately define the relevant action-angle variables. First, we note that solutions to the equation

$$H\left(\zeta, \bar{\zeta}; \eta, \bar{\eta}, \omega\right) = E \tag{14}$$

are related to that of

$$H(\zeta, \bar{\zeta}) = E \tag{15}$$

by the group transformation

$$\zeta \to \zeta(\zeta, \eta, \omega). \tag{16}$$

Let $C(\eta,\bar{\eta},\omega)$ (resp. C) be a curve defined by equation (14) (resp. equation (15)). Due to the transformation property

$$\frac{\bar{\zeta}(\zeta,\eta,\omega)\mathrm{d}\zeta(\zeta,\eta,\omega)-\zeta(\zeta,\eta,\omega)\mathrm{d}\bar{\zeta}(\zeta,\eta,\omega)}{1-|\zeta(\zeta,\eta,\omega)|^2} = \frac{\bar{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}+\frac{\bar{\eta}\mathrm{e}^{2\mathrm{i}\omega}\mathrm{d}\zeta}{1+\bar{\eta}\zeta\mathrm{e}^{2\mathrm{i}\omega}}-\frac{\eta\mathrm{e}^{-2\mathrm{i}\omega}\mathrm{d}\bar{\zeta}}{1+\eta\bar{\zeta}\mathrm{e}^{-2\mathrm{i}\omega}} \tag{17}$$

we have

$$\oint\limits_{C(\eta,\bar{\eta},\omega)}\frac{\bar{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}=\oint\limits_{C}\frac{\bar{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}. \tag{18}$$

We conclude that

$$I=I(\zeta,\bar{\zeta};\eta,\bar{\eta},\omega)=I\left(\zeta(\zeta;-\eta;-\omega),\bar{\zeta}(\zeta;-\eta;-\omega)\right). \tag{19}$$

The generating function $S(\zeta,\bar{\zeta}I;\eta,\bar{\eta},\omega)$ reads

$$S(\zeta,\bar{\zeta},I;\eta,\bar{\eta},\omega)=\frac{\mathrm{i}a}{2}\int\limits_{\zeta(\zeta_0,\eta,\omega)}^{\zeta}\frac{\bar{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}. \tag{20}$$

By virtue of equation (17) we have

$$S(\zeta,\bar{\zeta},I;\eta,\bar{\eta},\omega)=S(\zeta(\zeta;-\eta;-\omega),\bar{\zeta}(\zeta;-\eta;-\omega),I)+\text{terms not containing } I. \tag{21}$$

Therefore

$$\varphi=\varphi(\zeta,\bar{\zeta};\eta,\bar{\eta},\omega)=\varphi(\zeta(\zeta;-\eta;-\omega),\bar{\zeta}(\zeta;-\eta;-\omega)) \tag{22}$$

is the relevant action variable. Equations (19), (22) can be solved in terms of $\zeta,\bar{\zeta}$ to yield

$$\begin{aligned}\zeta&=\zeta(\zeta(\varphi,I),\eta,\omega)\\ \bar{\zeta}&=\bar{\zeta}(\zeta(\varphi,I),\eta,\omega)\end{aligned} \tag{23}$$

where $\zeta(\varphi,I)$, $\bar{\zeta}(\varphi,I)$ are the solutions corresponding to $\eta=\omega=0$.

Let us now assume that the parameters η, $\bar{\eta}$ and ω depend on time. It is straightforward (cf section 1) to calculate the relevant Hannay angles. The result reads

$$\Delta\varphi(I,C)=-\frac{\mathrm{i}a}{2}\frac{\partial}{\partial I}\oint\limits_{C}\frac{1}{2\pi}\int\limits_0^{2\pi}\mathrm{d}\varphi\left(\frac{\bar{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\bar{\zeta}}{1-|\zeta|^2}\right) \tag{24}$$

where C is now a closed curve in parameter space $(\eta,\bar{\eta},\omega)$, $\zeta,\bar{\zeta}$ are given by equations (23) and

$$\begin{aligned}\mathrm{d}\zeta&=\frac{\partial\zeta}{\partial\eta}\mathrm{d}\eta+\frac{\partial\zeta}{\partial\bar{\eta}}\mathrm{d}\bar{\eta}+\frac{\partial\zeta}{\partial\omega}\mathrm{d}\omega\\ \mathrm{d}\bar{\zeta}&=\frac{\partial\bar{\zeta}}{\partial\eta}\mathrm{d}\eta+\frac{\partial\bar{\zeta}}{\partial\bar{\eta}}\mathrm{d}\bar{\eta}+\frac{\partial\bar{\zeta}}{\partial\omega}\mathrm{d}\omega.\end{aligned} \tag{25}$$

In what follows we assume that the initial Hamiltonian depends on $|\zeta|$ only, $H=H(|\zeta|)$. By virtue of equation (9) we have

$$I=\frac{|a||\zeta|^2}{1-|\zeta|^2}\qquad H(|\zeta|)=E. \tag{26}$$

On the other hand, the angle variable is equal to $\arg\zeta$

$$\varphi = \arg\zeta\,. \tag{27}$$

Equations (23)–(27) then imply the following formula for the Hannay angle:

$$\Delta\varphi(I, C) = -\mathrm{i}\,\mathrm{sgn}\,a \oint\limits_C \frac{\bar\eta \mathrm{d}\eta - \eta \mathrm{d}\bar\eta}{1-|\eta|^2}\,. \tag{28}$$

A nice feature of this formula is that the Hannay angle is expressed in terms of Cartan forms on $SU(1,1)/U(1)$, where $U(1)$ consists of those elements of $SU(1,1)$ which commute with the Hamiltonian. The angle is completly independent on the particular form of the function $H(\cdot)$.

3. Coherent states and quantum dynamics

In this section we consider the quantum theories based on the unitary irreducible representations of $SU(1,1)$ (the $SU(1,1)$ inavriant quantum systems were investigated in numerous papers; see, for example, [12]). The classical limit in the sense of Yaffe [8] is analysed and the relationship between Hannay's angle and Berry's phase, found by Berry [2] in the case of Heisenberg group, is re-established.

It is well known that $SU(1,1)$ possesses few series representations. We shall consider here only the discrete series; it is sufficient to consider $D_k^{(+)}$ (the treatment of $D_k^{(-)}$ is analogous). The space of states is spanned by the vectors $|k, k+m\rangle$, $m = 0, 1, \ldots,$ $k = 1, \frac{3}{2}, 2, \ldots$; here $k+m$ are the eigenvalues of K_0,

$$K_0|k, k+m\rangle = (k+m)|k, k+m\rangle\,. \tag{29}$$

The eigenvectors $|k, k+m\rangle$ are obtained from $|k,k\rangle$ by succesive application of K_+

$$|k, k+m\rangle = \left(\frac{\Gamma(2k)}{m!\Gamma(2k+m)}\right)^{1/2} (K_+)^m\,|k,k\rangle\,. \tag{30}$$

The Hilbert space carrying the representation $D_k^{(+)}$ is our space of states of quantum mechanical system; the observables are constructed from the generators K_i.

The classical limit is defined as follows: $\hbar \to 0$, $k \to \infty$, $k\hbar$ = constant. This classical limit will be constructed according to Yaffe's receipe. First, we define the coherent states [11, 12]

$$|\zeta, \psi\rangle = \mathrm{e}^{\xi K_+ - \bar\xi K_-}\mathrm{e}^{\mathrm{i}\psi K_0}|k,k\rangle = \mathrm{e}^{\mathrm{i}k\psi}\mathrm{e}^{\xi K_+ - \bar\xi K_-}|k,k\rangle \equiv \mathrm{e}^{\mathrm{i}k\psi}|\zeta\rangle\,. \tag{31}$$

In order to find a well defined classical counterpart of our system the assumptions made in Yaffe's paper have to be verifed. It is easy to check that

(i) The states $|\zeta,\psi\rangle$, $|\zeta',\psi'\rangle$ are classically equivalent in the sense of Yaffe's, $|\zeta,\psi\rangle \sim |\zeta',\psi'\rangle$, if and only if $\zeta = \zeta'$.

(ii) The operators $\hbar K_i$ and all their functions are classical operators in the sense of Yaffe.

Let us find the symbols of the operators K_i. One can easily check that

$$\begin{aligned}
\langle\zeta|K_0|\zeta\rangle &= k\frac{1+|\zeta|^2}{1-|\zeta|^2}\\
\langle\zeta|K_+|\zeta\rangle &= 2k\frac{\bar\zeta}{1-|\zeta|^2}\\
\langle\zeta|K_-|\zeta\rangle &= 2k\frac{\zeta}{1-|\zeta|^2}.
\end{aligned} \tag{32}$$

An arbitrary constant a appearing in the definition of the Poisson bracket (5) can now be fixed by demanding that the usual quantization rule $\{,\} \to \frac{1}{i\hbar}[,]$ preserves Lie algebra relations; one obtains

$$a = 2k\hbar\,. \tag{33}$$

The correspondence between operators and their symbols provides a proper relation between the quantum and classical theories; let us show, for example, that the spectrum of K_0 can be found by semiclassical quantization method. Let $H = \hbar\omega K_0$ be a Hamiltonian. The curve $H(\zeta, \bar{\zeta}) = E$ is given by the formula

$$|\zeta|^2 = \frac{E - \hbar\omega k}{E + \hbar\omega k}\,. \tag{34}$$

Therefore, by virtue of equation (9),

$$I = \frac{1}{\omega}(E - \hbar\omega k)\,. \tag{35}$$

Using the semiclassical quantization rule

$$I = m\hbar \qquad m \text{ integer} \tag{36}$$

one recovers the spectrum of K_0. Note that the semiclassical formula gives an exact answer; moreover, no Keller–Maslov index is necessary. This result may be also explained as folows. Let us calculate the scalar product

$$\langle k, k+m|\zeta\rangle = \left(1 - |\zeta|^2\right)^k \left[\frac{\Gamma(m+2k)}{\Gamma(2k)m!}\right]^{1/2} \zeta^m\,. \tag{37}$$

The dominant contribution in the limit $\hbar \to 0$, $k \to \infty$, $k\hbar$ = constant; $m\hbar$ = constant comes from the state $|k, k+m\rangle$ with

$$m = \frac{2k|\zeta|^2}{1 - |\zeta|^2} \equiv \frac{I}{\hbar} \tag{38}$$

(actually, it is sufficient to take the limit $m \to \infty$, $m\hbar$ = constant; see below).

Let us now consider the family of Hamiltonians obtained from the initial Hamiltonian H which is the function of K_0 only. More precisely

$$\hat{H}(\eta, \bar{\eta}, \omega) = U(\eta, \bar{\eta}, \omega)\hat{H}(K_0)U^+(\eta, \bar{\eta}, \omega)\,. \tag{39}$$

Actually, due to equation (1) we conclude that $\hat{H}$ does not depend on ω. We assume also that the spectrum of $\hat{H}$ is non-degenerate. $\hat{H}(\eta, \bar{\eta})$ can be made time-dependent by making the parameter η time-dependent. According to the general formula given in [13] it is easy to calculate the corresponding Berry phase. Due to the conditions

$$\langle k, k+m|K_\pm|k, k+m\rangle = 0 \tag{40}$$

the Berry phase is expressed in terms of the Cartan form λ. Namely, the following formula holds

$$\gamma_m = (k+m)\oint_C \frac{\mathrm{i}(\bar{\eta}\,\mathrm{d}\eta - \eta\,\mathrm{d}\bar{\eta})}{1 - |\eta|^2}\,. \tag{41}$$

In order to compare the above expression with the one for Hannay's angle let us note that the classical Hamiltonian

$$H(\zeta, \bar{\zeta}; \eta, \bar{\eta}, \omega) = \langle k, k|U^+(\zeta, \bar{\zeta})U(\eta, \bar{\eta}, \omega)H(K_0)U^+(\eta, \bar{\eta}, \omega)U(\zeta, \bar{\zeta})|k, k\rangle \tag{42}$$

has the form (13) with the initial Hamiltonian depending only on $|\zeta|^2$. The relevant Hannay's angle is given by equation (28). From equations (28) and (41) we conclude that the Berry relation

$$\varphi = -\frac{\partial \gamma_m}{\partial m} \tag{43}$$

is obeyed.

Let us conclude with few remarks concerning the most relevant $SU(1,1)$ systems—the harmonic oscillator [14]. The Hamiltonian of generalized harmonic oscillator (with the term $pq+qp$ present) can be expressed in terms of $SU(1,1)$ generators. Therefore we can apply the formalism presented above to calculate both the Hannay's angle and Berry's phase. The results obviously agree with those obtained in [2]. This provides an alternative approach for the system which can be also described in terms of standard phase space (q, p). However, in this case the classical limit $\hbar \to 0$ is attained within fixed representation of $SU(1,1)$ (or, rather, its covering group).

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